Meta-Optimization

of

Particle Swarm Optimization

by

Magnus Erik Hvass Pedersen

What is Optimization?

- An optimization problem is a function f mapping candidate solutions to a fitness measure:

$$f: \mathbb{R}^n \rightarrow \mathbb{R}$$

- The optimal solution $\vec{z} \in \mathbb{R}^n$ has the best fitness and therefore satisfies the equation (minimization):

$$\forall \vec{y} \in \mathbb{R}^n: \; f(\vec{z}) \leq f(\vec{y})$$

- Searching for the optimal solution can be done by iteratively following the gradient ∇f
- If f is not differentiable or the gradient is unknown then f is a 'black-box' and we can use heuristic optimization.

Particle Swarm Optimization (PSO)

Pseudo-code:

- Initialize each particle $\vec{x}$ to a random position in the search-space and give it a random velocity $\vec{v}$.
- Until a termination criterion is met, update the velocity and position for each particle using the formulas:

$$\vec{v} \leftarrow \omega \vec{v} + \phi_p r_p (\vec{p} - \vec{x}) + \phi_g r_g (\vec{g} - \vec{x})$$

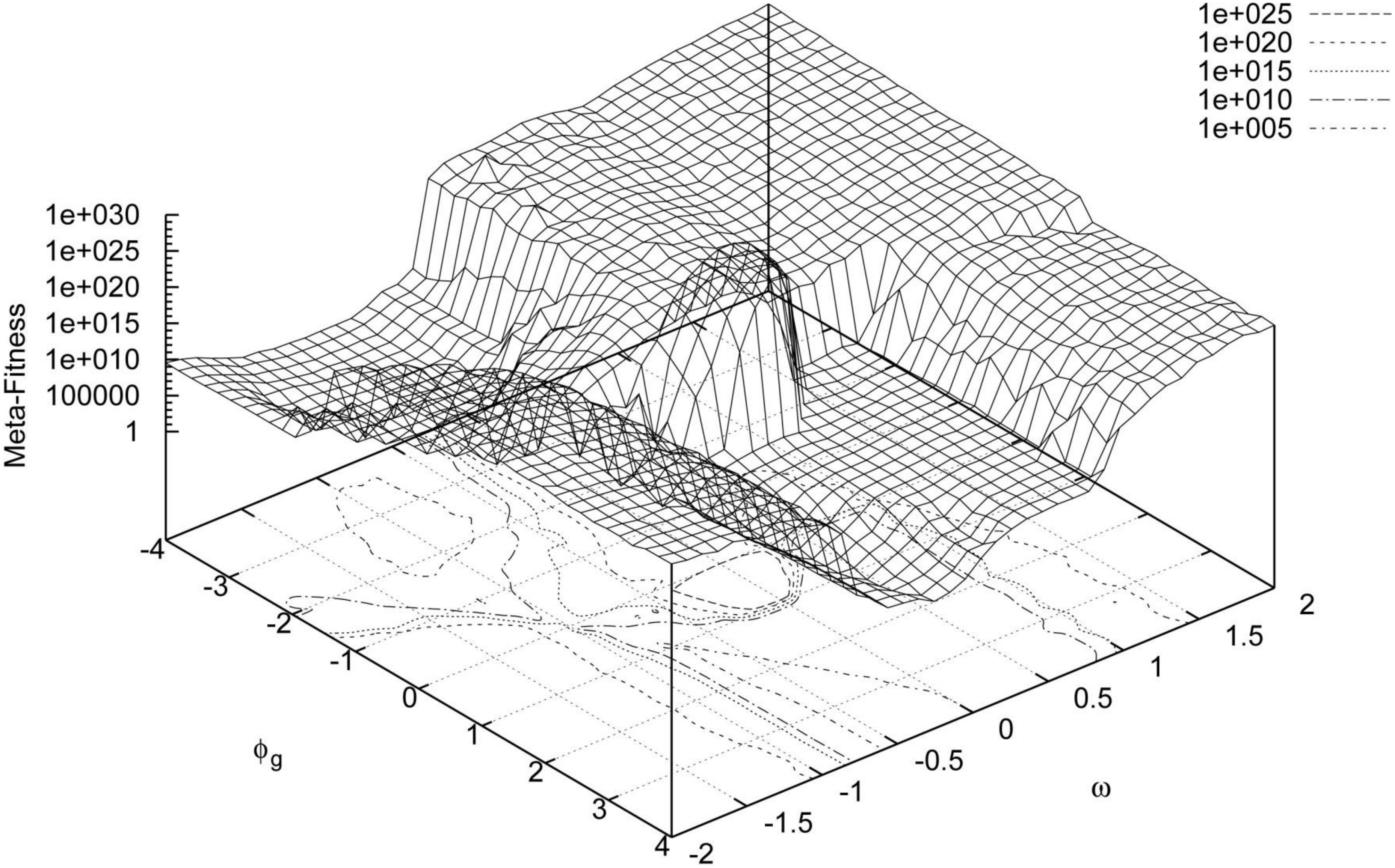
$$\vec{x} \leftarrow \vec{x} + \vec{v}$$

- Where r_p and r_g are random numbers, $\vec{p}$ is the particle's best known position and $\vec{g}$ is the swarm's best position.

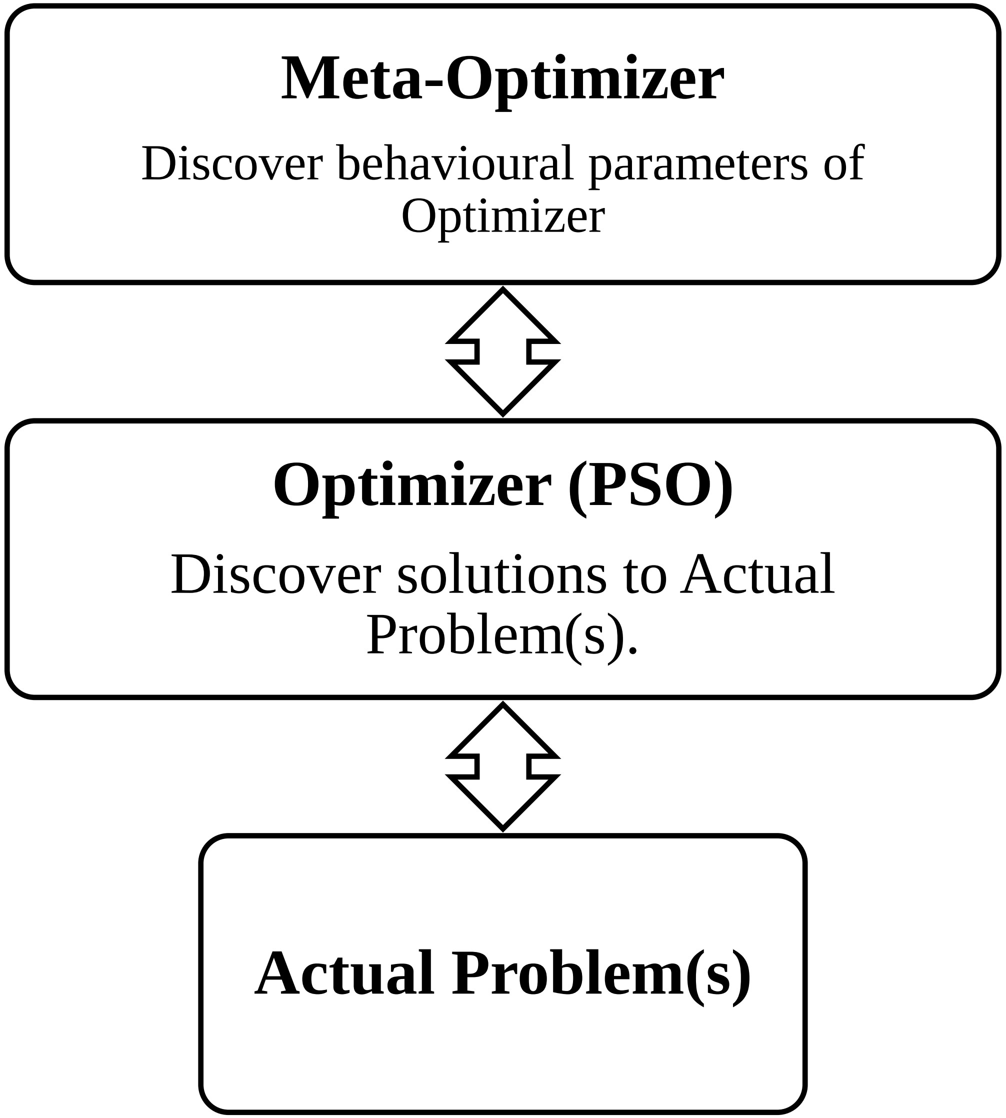
Behavioural Parameters

- PSO parameters: S (number of particles), ω, ϕ_p and ϕ_g
- Greatly influence optimization performance.
- May be determined by trial-and-error (hand-tuning), adaptive and self-adaptive schemes.
- As the number of parameters increases, the number of possible combinations increases exponentially, this is the Curse of Dimensionality for the parameter space.
- We need an efficient way of searching for the best parameters for an optimizer, that is, we need an additional layer of optimization.

Search-Space of PSO Parameters



Meta-Optimization



Meta-Optimization

Pseudo-code:

- Initialize the meta-optimizer with a random choice of behavioural parameters for the PSO optimizer.
- Repeat the following a number of times:
    - Conduct optimization runs with PSO using the given choice of behavioural parameters.
    - Sum the results of the PSO runs to form a meta-fitness measure of its performance.
    - Modify the behavioural parameters using the optimization methodology of the meta-optimizer.

Meta-Optimization

Challenges and solutions:

- Very time-consuming because each meta-level iteration consists of evaluating the performance of the optimizer.
- The LUS method is used as meta-optimizer because it is simple yet often discovers optima within a few iterations; especially for smoother problems such as the search-spaces of behavioural parameters.
- Pre-emptive Fitness Evaluation aborts a meta-fitness evaluation once it becomes known that it does not lead to an improvement.
- Together these limit time-usage to minutes or hours depending on problem settings.

Meta-Optimized Parameters

- Good parameters depend on problem settings used in meta-optimization.
- For example, meta-optimization using 12 benchmark problems in 20 dimensions and allowing 40.000 fitness evaluations finds these PSO parameters to be good:

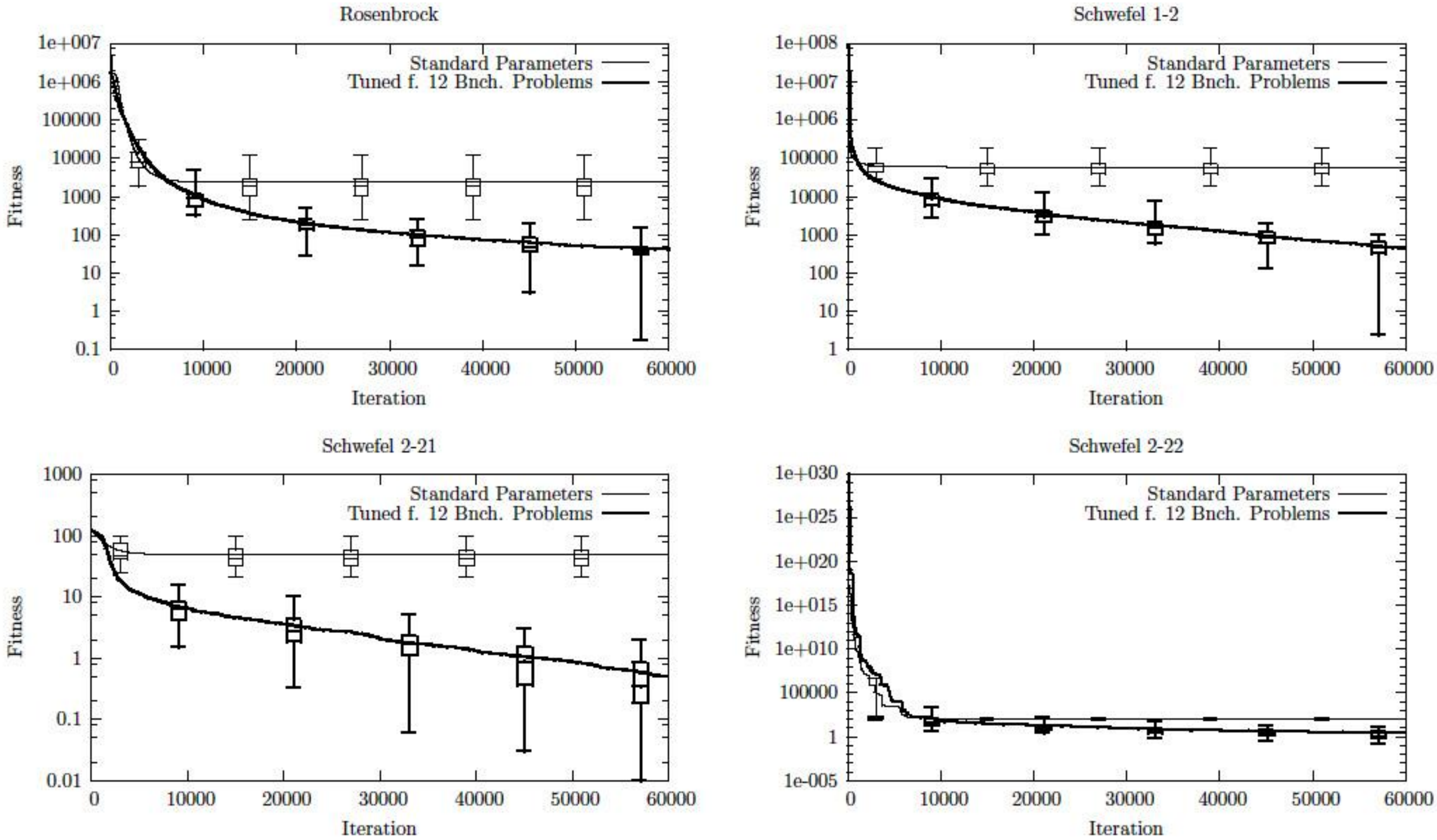
$$S = 69 \quad \omega = -0.44 \qquad \phi_p = -0.27 \qquad \phi_g = 3.40$$

- Compare to hand-tuned parameters:

$$S = 50 \quad \omega = 0.729 \qquad \phi_p = \phi_g = 1.4944$$

- Note that the tuned parameters ω and ϕ_p are negative, which contradicts conventional wisdom about how PSO works.

Performance Comparison



References

Good Parameters for Particle Swarm Optimization

Tuning & Simplifying Heuristical Optimization

SwarmOps source-code C, C#, Java

www.hvass-labs.org