Kelly vs. Markowitz Portfolio Optimization

by

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Kelly vs. Markowitz Portfolio Optimization

Markowitz portfolios do NOT minimize risk even if given the true probability distribution of asset returns.

Kelly portfolios maximize average long-term returns if given the true return distribution. But Kelly portfolios have other problems.

Portfolio Rate of Return

The portfolio rate of return is the weighted sum of asset rates of return:

$$Portfolio\ Rate\ of\ Return = \sum_i Asset\ Weight_i \cdot Asset\ Rate\ of\ Return_i$$

The weights must sum to one: $\sum_i Asset\ Weight_i = 1$

In long-only portfolios the weights must be non-negative.

Markowitz Portfolio Optimization

Find asset weights that maximize mean return and minimize variance.

Maximize: $E[Portfolio\ Rate\ of\ Return]$

Minimize: $Var[Portfolio\ Rate\ of\ Return]$

See previous talk for details.

Variance is Not Risk (Negative Returns)

Simple example:

Asset	Possible Returns			Mean	Stdev
A	(4%)	(5%)	(6%)	(5%)	1%
B	5%	10%	15%	10%	5%

Anti-correlated with coefficient -1.

Minimum variance portfolio:

Asset A Weight	5/6
Asset B Weight	1/6
Portfolio Mean	(2.5%)
Portfolio Stdev	0%

So the minimum-variance (“minimum-risk”) portfolio has a loss of (2.5%) for all possible outcomes, while Asset B has a gain of either 5%, 10% or 15%. Clearly Asset B is a better investment.

Variance is Not Risk (Positive & Overlapping Returns)

Positive Returns:

Asset	Possible Returns			Mean	Stdev
A	3%	2%	1%	2%	1%
B	5%	10%	15%	10%	5%

Minimum variance portfolio:

Asset A Weight	5/6
Asset B Weight	1/6
Portfolio Mean	3.3%
Portfolio Stdev	0%

Overlapping Returns:

Asset	Possible Returns			Mean	Stdev
A	6%	5%	4%	5%	1%
B	5%	10%	15%	10%	5%

Minimum variance portfolio:

Asset A Weight	5/6
Asset B Weight	1/6
Portfolio Mean	5.8%
Portfolio Stdev	0%

Kelly Portfolio Optimization

The Kelly Criterion is defined as the mean logarithmic growth:

$$Kelly\ Criterion = E[\log{(1 + Portfolio\ Rate\ of\ Return)}]$$
$$= E\left[\log\left(1 + \sum_i Asset\ Weight_i \cdot Asset\ Rate\ of\ Return_i\right)\right]$$

The objective is to find the weights that maximize the Kelly criterion. This can be done with a numerical optimization method.

Kelly Portfolio Optimization (Example 1)

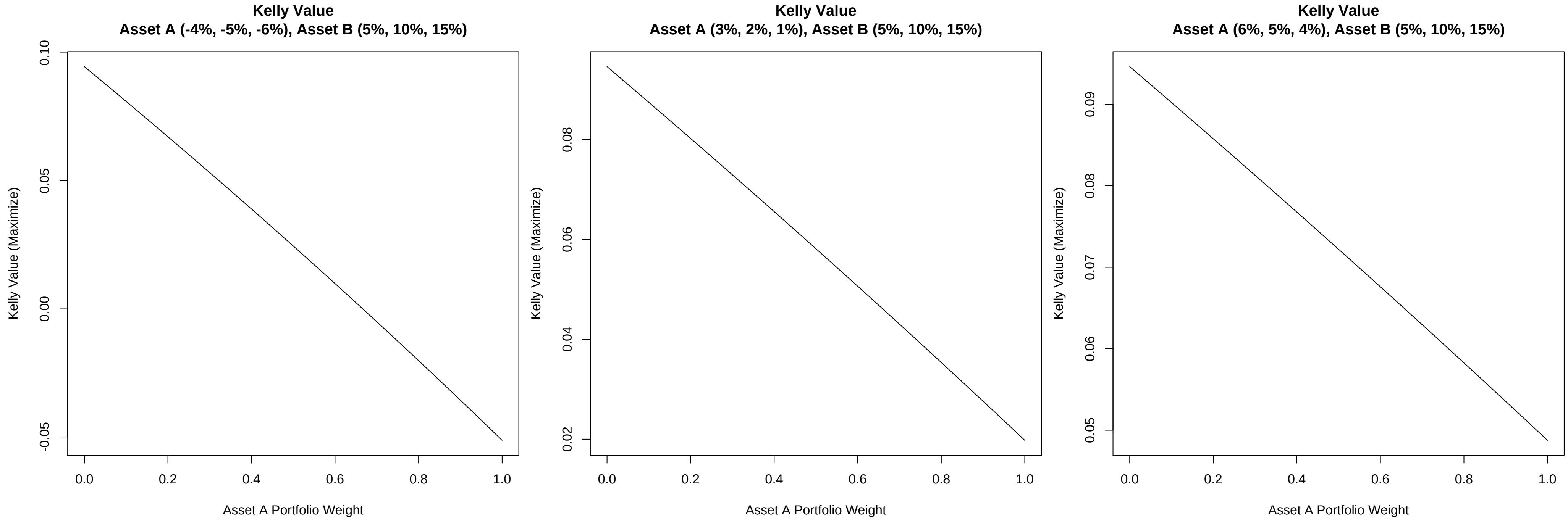
Asset A returns: (4%), (5%) or (6%).

Asset B returns: 5%, 10% or 15%.

Find the weights for Asset A and Asset B that maximize:

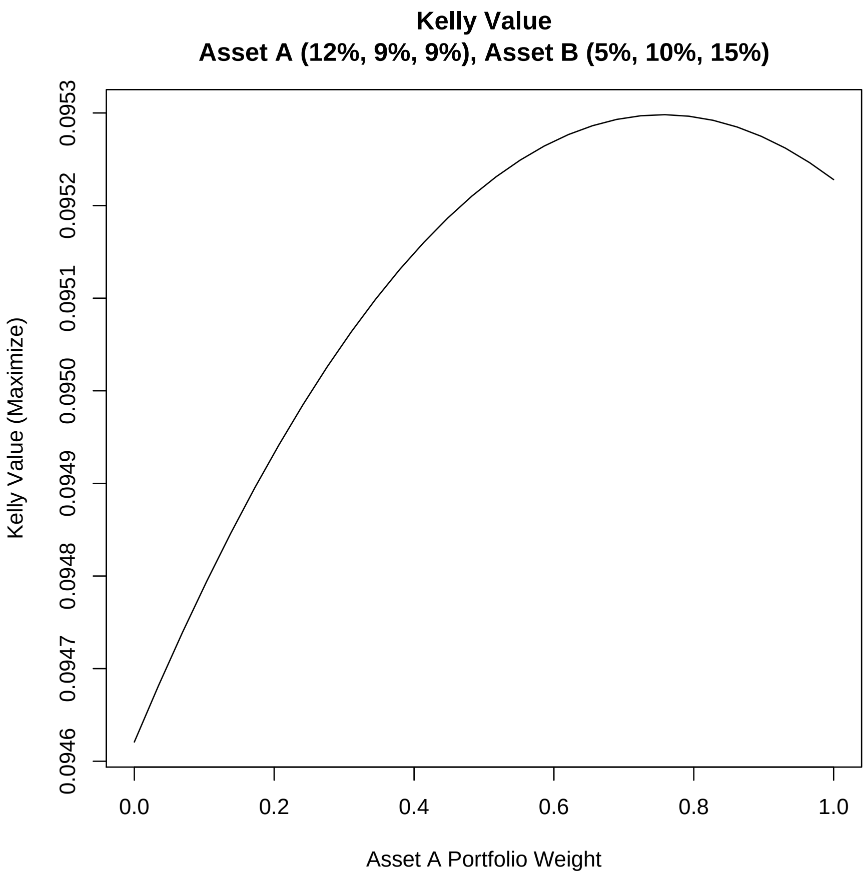
$$Kelly\ Criterion =$$
$$= E[\log(1 + Asset\ A\ Weight \cdot (-4\%, -5\%, -6\%) + Asset\ B\ Weight \cdot (5\%, 10\%, 15\%))]$$

Kelly Portfolio Optimization (Example 1)



Maximum is when Asset A Weight = 0 and Asset B Weight = 1.

Kelly Portfolio Optimization (Example 2)



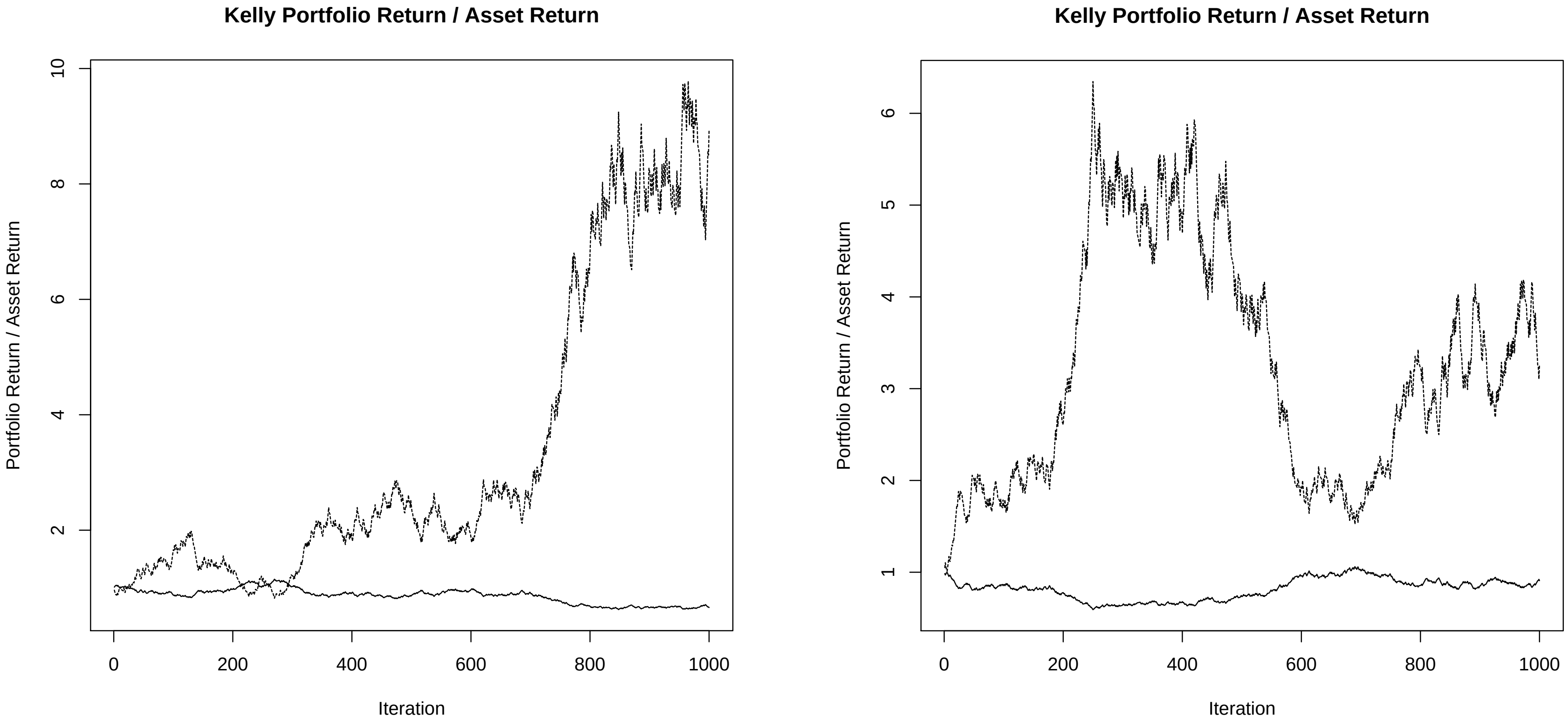
Asset A Returns:
12%, 9% or 9%

Asset B Returns:
5%, 10% or 15%

Max weights:
Asset A = 0.756
Asset B = 0.244

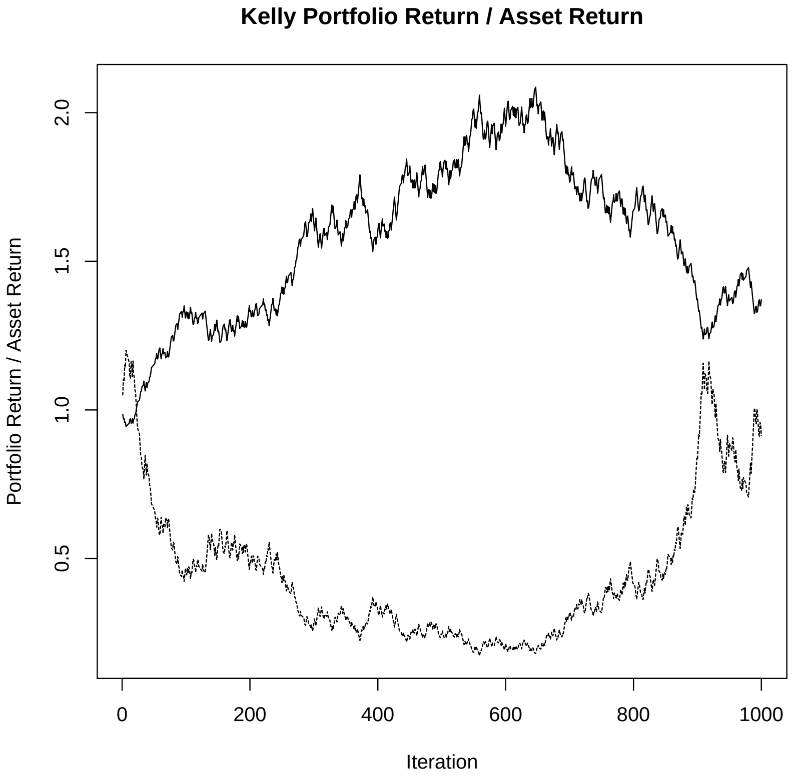
Portfolio Returns:
10.3%
9.2%
10.5%

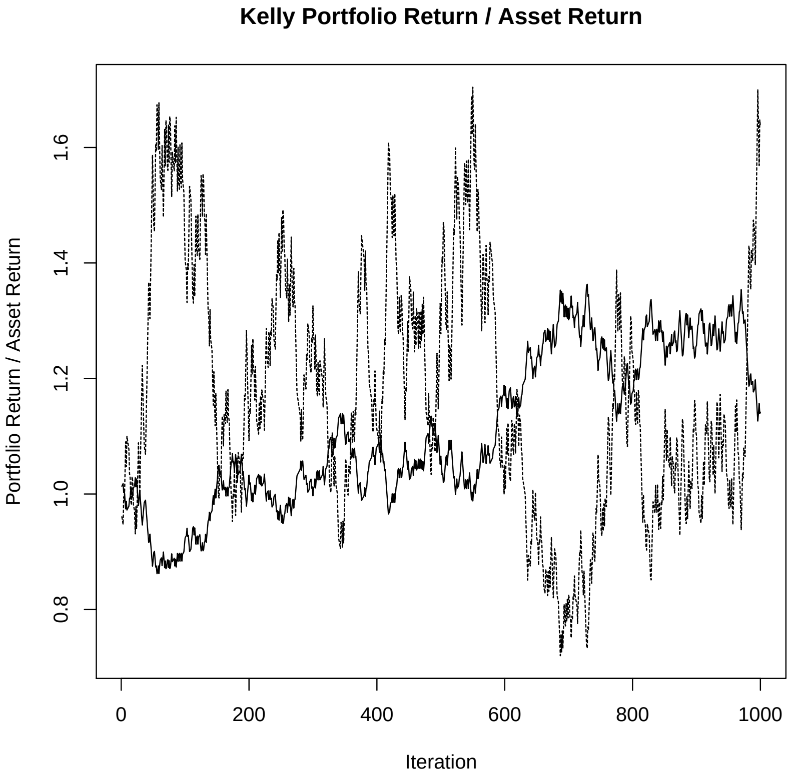
Simulated Compounded Returns (Example 2)



Solid lines show Kelly Portfolio / Asset A. Dashed lines show Kelly Portfolio / Asset B.

Simulated Compounded Returns (Example 2)





Solid lines show Kelly Portfolio / Asset A. Dashed lines show Kelly Portfolio / Asset B.

Conclusion for Markowitz

- Markowitz (Mean-Variance) portfolios do not maximize return and minimize risk as commonly believed, even when given the true probability distribution of returns.
- But Markowitz portfolios are diversified which may give an illusion of safety.

Conclusion for Kelly

- Kelly portfolio optimization does what it is supposed to: Favours assets with better return distributions.
- Kelly portfolios may underperform in the “short run” but will have the best performance on average in the “long run”.
- Kelly portfolios are often concentrated in few assets. So if the return distributions are incorrect then Kelly overweighs the wrong assets.
- Diversification of Kelly can be enforced by limiting the asset weights.

Further Reading

This lecture is based on:

- **Portfolio Optimization and Monte Carlo Simulation**
- **Source-code for R**
- **MS Excel Spreadsheet**

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Available on the internet:

www.Hvass-Labs.Org