

Portfolio Variance is Not Risk

by

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Mean-Variance Portfolio Optimization

Combine assets so as to maximize mean return and minimize variance.

Aka. Markowitz portfolio optimization or Modern Portfolio Theory.

Variance is commonly believed to measure investment risk.

This is FALSE as can be demonstrated with a few simple examples.

Portfolio Return

The rate of return on a portfolio is the weighted sum of asset returns:

$$\textit{Portfolio Return} = \sum_i \textit{Weight}_i \cdot \textit{Asset Return}_i$$

The weights must sum to one:

$$\sum_i \textit{Weight}_i = 1$$

In long-only portfolios the weights must be non-negative: $\textit{Weight}_i \geq 0$

Mean and Variance of Portfolio Return

Mean portfolio return:

$$E[\textit{Portfolio Return}] = \sum_i \textit{Weight}_i \cdot E[\textit{Asset Return}_i]$$

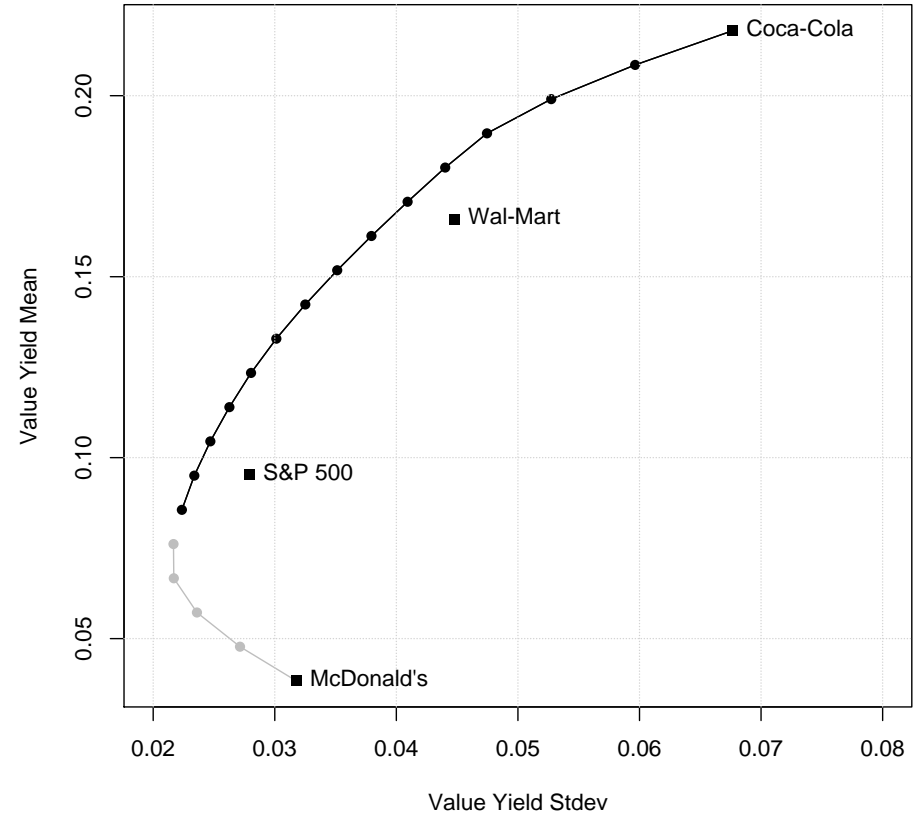
Variance of portfolio return (standard deviation is square root of this):

$$\begin{aligned} \textit{Var}[\textit{Portfolio Return}] \\ = \sum_{i,j} \textit{Weight}_i \cdot \textit{Weight}_j \cdot \textit{Cov}[\textit{Asset Return}_i, \textit{Asset Return}_j] \end{aligned}$$

Mean-Variance Efficient Frontier

- The feasible set contains all possible portfolios by changing the weights.
- The efficient frontier maximizes the mean and minimizes the variance.
- Efficient frontier is computed from the asset means and covariance.

Efficient Frontier (Long Only, 10-Year Holding)



Variance is Not Risk (Negative Returns)

Simple example:

Asset	Possible Returns			Mean	Stdev
A	(4%)	(5%)	(6%)	(5%)	1%
B	5%	10%	15%	10%	5%

Anti-correlated with coefficient -1.

Minimum variance portfolio:

Asset A Weight	5/6
Asset B Weight	1/6
Portfolio Mean	(2.5%)
Portfolio Stdev	0%

So the minimum-variance (“minimum-risk”) portfolio has a loss of (2.5%) for all possible outcomes, while Asset B has a gain of either 5%, 10% or 15%. Clearly Asset B is a better investment.

Variance is Not Risk (Positive Returns)

Simple example:

Asset	Possible Returns			Mean	Stdev
A	3%	2%	1%	2%	1%
B	5%	10%	15%	10%	5%

Anti-correlated with coefficient -1.

Minimum variance portfolio:

Asset A Weight	5/6
Asset B Weight	1/6
Portfolio Mean	3.3%
Portfolio Stdev	0%

So the minimum-variance (“minimum-risk”) portfolio has a gain of 3.3% for all possible outcomes, while Asset B has a gain of either 5%, 10% or 15%. Clearly Asset B is a better investment.

Variance is Not Risk (Overlapping Returns)

Simple example:

Asset	Possible Returns			Mean	Stdev
A	6%	5%	4%	5%	1%
B	5%	10%	15%	10%	5%

Anti-correlated with coefficient -1.

Minimum variance portfolio:

Asset A Weight	5/6
Asset B Weight	1/6
Portfolio Mean	5.8%
Portfolio Stdev	0%

So the minimum-variance (“minimum-risk”) portfolio has a gain of 5.8% for all possible outcomes, while Asset B has a gain of either 5%, 10% or 15%. Asset B is a better investment with probability 2/3 and is only slightly worse than min-variance portfolio with probability 1/3.

Why is Asset A in the Minimum-Variance Portfolio?

- Asset A has a low standard deviation of 1% while Asset B has 5%.
- The asset returns are anti-correlated so combining them lowers the standard deviation (and variance).
- The mean-variance efficient frontier is optimized for low variance.
- The minimum-variance portfolio is on the efficient frontier.

Conclusion: Variance is Not Risk

- Variance measures the spread of possible returns.
- Variance doesn't take the probability of loss into account.
- Variance doesn't take the joint probability distributions into account.
- This also holds for other distributions e.g. the Normal distribution.
- It even holds if we know the true distributions of asset returns.
- Mean-variance portfolios are not optimized for risk in the traditional sense of the word: “*the chance of injury or loss*”.

Further Reading

This lecture is based on the paper:

- [Portfolio Optimization and Monte Carlo Simulation](#)

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Available on the internet:

www.Hvass-Labs.Org