<u>Share Buyback Valuation</u> <u>Mathematical Analysis of Relative Value</u> <u>(Part 7)</u>

Magnus Erik Hvass Pedersen

by

Page 1/18

Why is Mathematical Analysis Important?

- The relative value of a share buyback can be plotted in a graph.
- This shows informally that the relative value is non-linear.
- But graphical plots can be deceiving and we cannot form valid scientific conclusions from just looking at a graphical plot.
- Mathematical analysis uses tools such as algebra and calculus to formally reason about the properties of mathematical functions.
- If the analysis is carried out correctly, then the results can be trusted as scientifically valid.

Value WITHOUT Share Buyback

... is the potential for dividend payouts; that is, the excess cash plus present value of future earnings available for dividend payouts:

$$v = Excess Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$
$$V = \frac{v \cdot (1 - TaxDividend)}{Shares}$$

Page 3/18

Value WITH Share Buyback

A share buyback reduces the cash available for dividends.

... and reduces the number of shares.

$$W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Page 4/18

Relative Value of Share Buyback

... is the value of a share buyback relative to a dividend payout:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}}$$

Page 5/18

Limitations of Graphical Plot

Seems to be:

- Non-linear in *Buyback*
 - Non-linear in v

But:

- Can we be sure of this?
- Are there any surprises?
- What if v is very large?
- How exactly does *W*/*V* behave?



Page 6/18

Varying the Intrinsic Value

How does the relative value change when we vary the intrinsic value *v* while holding *MarketCap* and *Buyback* constant?

Assume:

- Intrinsic value is positive: v > 0
- Buyback amount is between zero and market-cap:

 $MarketCap \geq Buyback \geq 0$

Page 7/18

Lower Limit of Relative Value, Varying *v*

200 %

Equilibrium 100 % As v approaches positive zero, 0% the limit of W/V is negative -100 % infinity: -200 % **Relative Value W/V** Buyback W $\lim_{v \to 0^+} \frac{1}{V}$ $= \lim_{v \to 0^+}$ Buyback MarketCan $-\infty$ 0 0.5

Intrinsic Value v

2

Page 8/18

1.5

Upper Limit of Relative Value, Varying *v*

As *v* approaches positive infinity, the relative value of a share buyback converges to:

$$\lim_{v \to \infty} \frac{W}{V} = \lim_{v \to \infty} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}}$$
$$= \frac{MarketCap}{MarketCap - Buyback}$$



Page 9/18

Derivative Function, Varying *v*

The derivative of W/V with regards to the variable v is denoted $\partial_v W/V$ which is the rate of change of W/V around a given v when the other variables *MarketCap* and *Buyback* are held constant:

$$\partial_{v} \frac{W}{V} = \partial_{v} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2}$$

This is continuous and monotonically decreasing for v > 0.

Page 10/18

Derivative Analysis, Varying v, Upper Limit

As v approaches positive zero, the limit of $\partial_v W/V$ is infinity:

$$\lim_{v \to 0^+} \partial_v \frac{W}{V} = \lim_{v \to 0^+} \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2} = +\infty$$

So near v = 0 the relative value of a share buyback W/V changes greatly when there is a small change in the value v and the other variables *MarketCap* and *Buyback* remain constant.

Derivative Analysis, Varying v, Lower Limit

As v approaches infinity, the limit of $\partial_v W/V$ is zero:

$$\lim_{v \to \infty} \partial_v \frac{W}{V} = \lim_{v \to \infty} \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2} = 0^+$$

As v increases, the relative value of a share buyback W/V changes less.

Page 12/18

Derivative Analysis, Varying v, Graphical Plot

Derivative is continuous and monotonically decreasing. The limits are:

$$\lim_{v \to 0^+} \partial_v \frac{W}{V} = +\infty$$

$$\lim_{v\to\infty}\partial_v\frac{W}{V}=0^+$$



Page 13/18



Page 14/18

Limits of Relative Value, Varying Buyback

If Buyback = 0 then W/V = 1.

As *Buyback* approaches *MarketCap* $\neq v$, the limit of *W*/*V* is either positive or negative infinity:

$$\lim_{Buyback \to MarketCap} \frac{W}{V} = \lim_{Buyback \to MarketCap} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \pm \infty$$

If MarketCap > v then limit is $-\infty$, if MarketCap < v then $+\infty$.

Page 15/18

Derivative Function, Varying Buyback

The derivative of W/V for the *Buyback* variable is the rate of change of W/V around a given *Buyback* when *MarketCap* and v are held constant:

$$\partial_{Buyback} \frac{W}{V} = \partial_{Buyback} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{MarketCap}{v}}{\frac{Buyback^2}{MarketCap} - 2 \cdot Buyback + MarketCap}$$

This is continuous and monotonic when *Buyback* goes from zero to *MarketCap*.

If Buyback = 0 then it is $(v - MarketCap)/(v \cdot MarketCap)$.

If *MarketCap* > v (overpriced shares) then it is decreasing and has limit $-\infty$.

If *MarketCap* < v (underpriced shares) then it is increasing and has limit + ∞ . Page 16/18

Conclusion

- The relative value of a share buyback is a non-linear function with regard to both the intrinsic value v and the buyback amount.
- Buyback of overpriced shares is much more destructive to long-term shareholder value than gains from buyback of underpriced shares.
- This effect is greatly magnified as the buyback amount increases.

Further Reading

This lecture is taken from the paper:

• The Value of Share Buybacks

Authored by Magnus Erik Hvass Pedersen.

Available on the internet:

www.Hvass-Labs.Org

Page 18/18