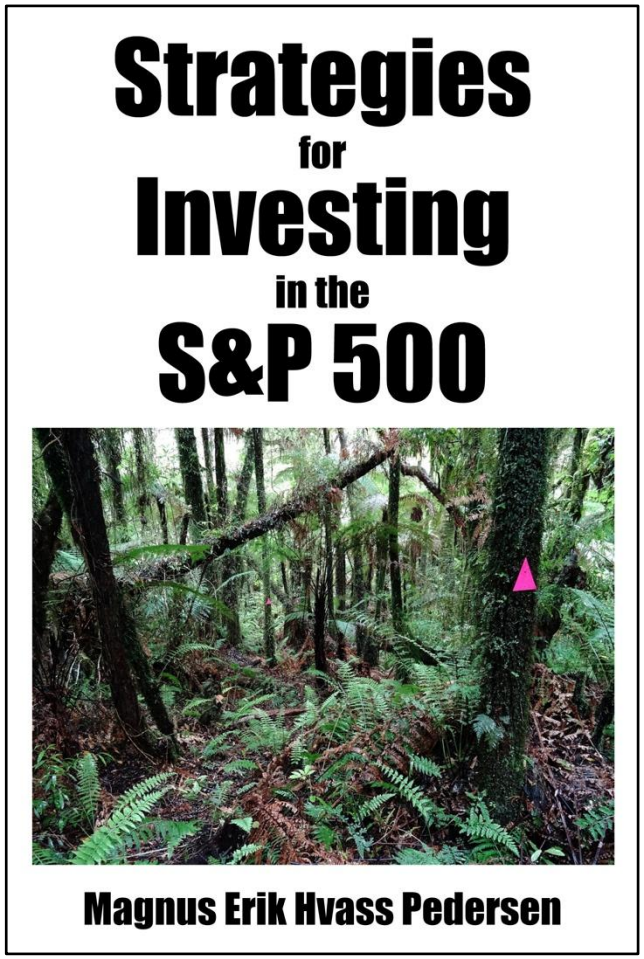

S&P 500 – Adaptive Rebalancing

(Part 3)

By

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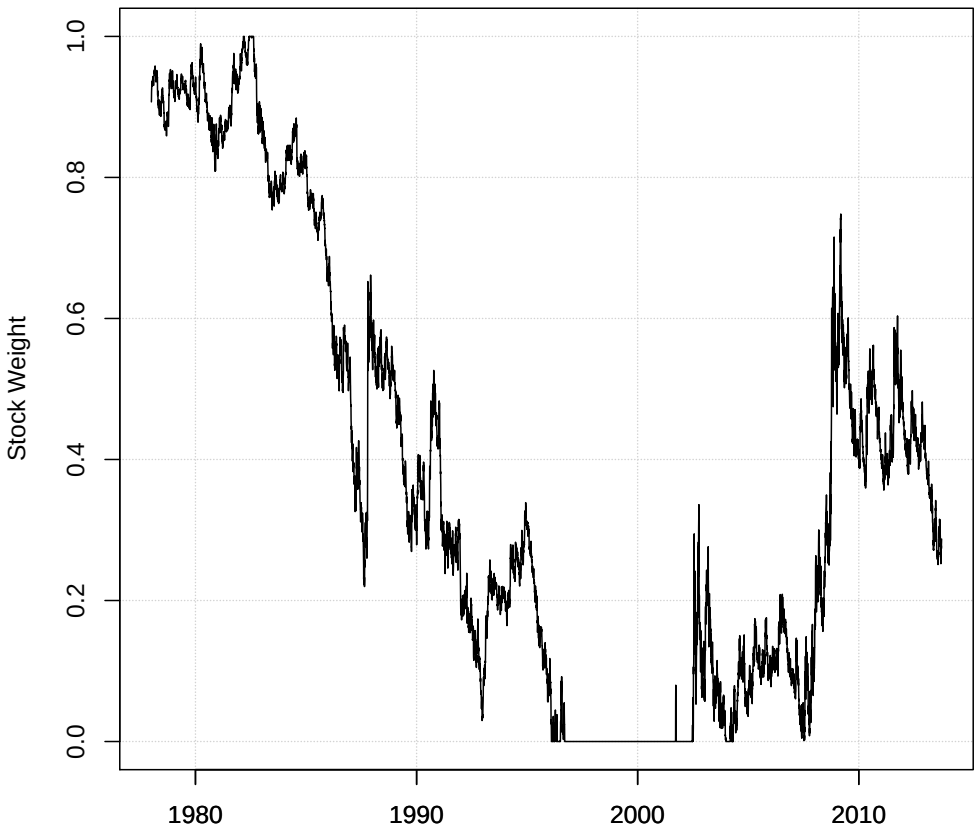
S&P 500 vs. US Government Bonds

- During the period 1978-2013 the average annualized return was almost 6% for US Government Bonds with 1 year maturity.
- The bond returns are guaranteed by the government of USA.
- The average annualized return for the S&P 500 was 11-13% depending on investment duration.
- But the S&P 500 was very volatile with a standard deviation over 17% for annual returns. The greatest annual gain was over 70%, the greatest annual loss was almost (50%).

Fixed vs. Adaptive Rebalancing

- Rebalance between S&P 500 and US Gov. Bonds to lower volatility.
- Each year the portfolio is rebalanced back to the desired allocation.
- Fixed rebalancing uses a predetermined allocation e.g. 50 / 50 or 25 / 75.
- It is simple but does not take the price-level of the S&P 500 into account.
- There is a relation between the P/Book (Price-To-Book ratio) and long-term returns of the S&P 500. (See another talk for more on this.)
- Adaptive rebalancing uses P/Book to adjust the portfolio allocation.

Stock Weight – Medium Risk Adaptive Rebalancing



The stock-weight is the part of the portfolio invested in the S&P 500. It is calculated using the P/Book of the S&P 500. The formula is:

Stock Weight = Limit(1.5 – 0.5 x P/Book)

Limited between zero and one.

Rebalancing is only done annually.

Example: Calculate the Stock Weight

On January 12, 1999 the P/Book was 4.64 so the stock-weight was:

$$\textit{Stock Weight} = \textit{Limit}(1.5 - 0.5 \times \textit{P/Book}) = \textit{Limit}(1.5 - 0.5 \times 4.64) = \textit{Limit}(-0.82) = 0$$

So the portfolio should be invested entirely in US Gov. Bonds.

The P/Book was high in several years. Then in 2003 it was 2.89:

$$\textit{Stock Weight} = \textit{Limit}(1.5 - 0.5 \times \textit{P/Book}) = \textit{Limit}(1.5 - 0.5 \times 2.89) = \textit{Limit}(0.06) \approx 0.06$$

In 2008 the P/Book was 2.52 so the stock-weight was:

$$\textit{Stock Weight} = \textit{Limit}(1.5 - 0.5 \times \textit{P/Book}) = \textit{Limit}(1.5 - 0.5 \times 2.52) = \textit{Limit}(0.24) \approx 0.24$$

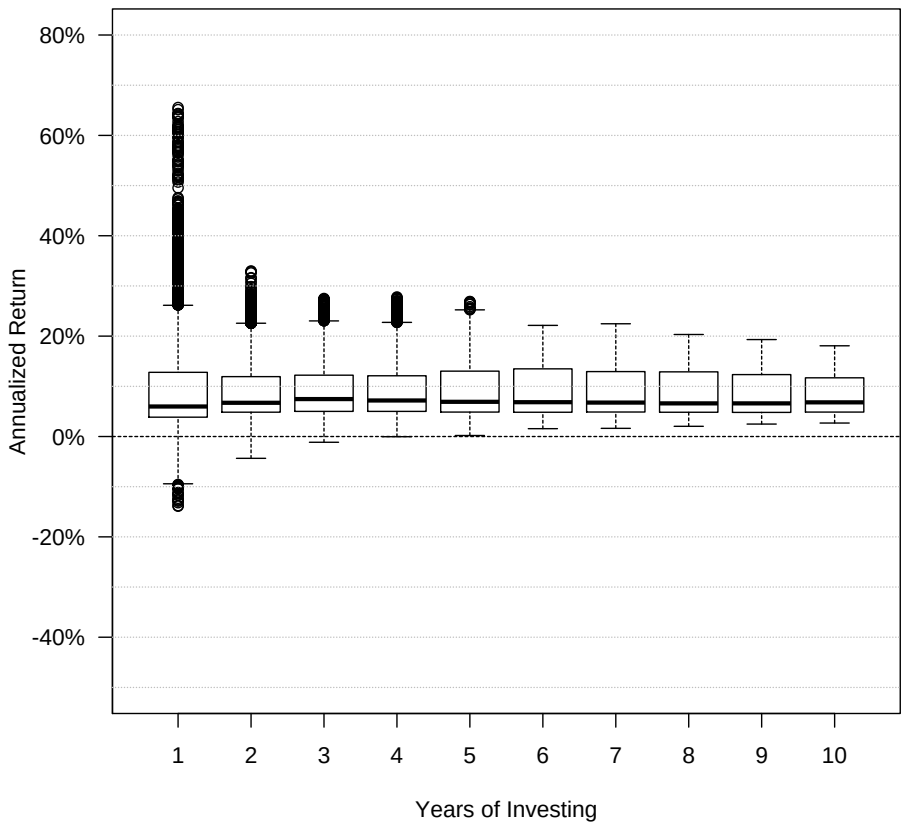
Example: Adaptive Rebalancing

- On January 12, 2008 the P/Book was 2.52 so stock weight was 0.24.
- Invest 24% of portfolio in S&P 500 and the rest in US. Gov. Bonds.
- From January 2008 to 2009 the S&P 500 lost about (38%).
- US Gov. Bonds yielded about 2.8% in that year.
- Return on the rebalanced portfolio from January 2008 to 2009 was:

$$\textit{Stock Weight} \times \textit{Stock Return} + (1 - \textit{Stock Weight}) \times \textit{Bond Return}$$

$$= 0.24 \times (38\%) + (1 - 0.24) \times 2.8\% \approx (7\%)$$

Medium Risk Rebal. – Annualized Return (1978-2013)



Back-test Medium Risk adaptive rebalancing for all possible starting dates and investment periods up to 10 years during 1978-2013.

Box-plot shows statistics for the annualized return.

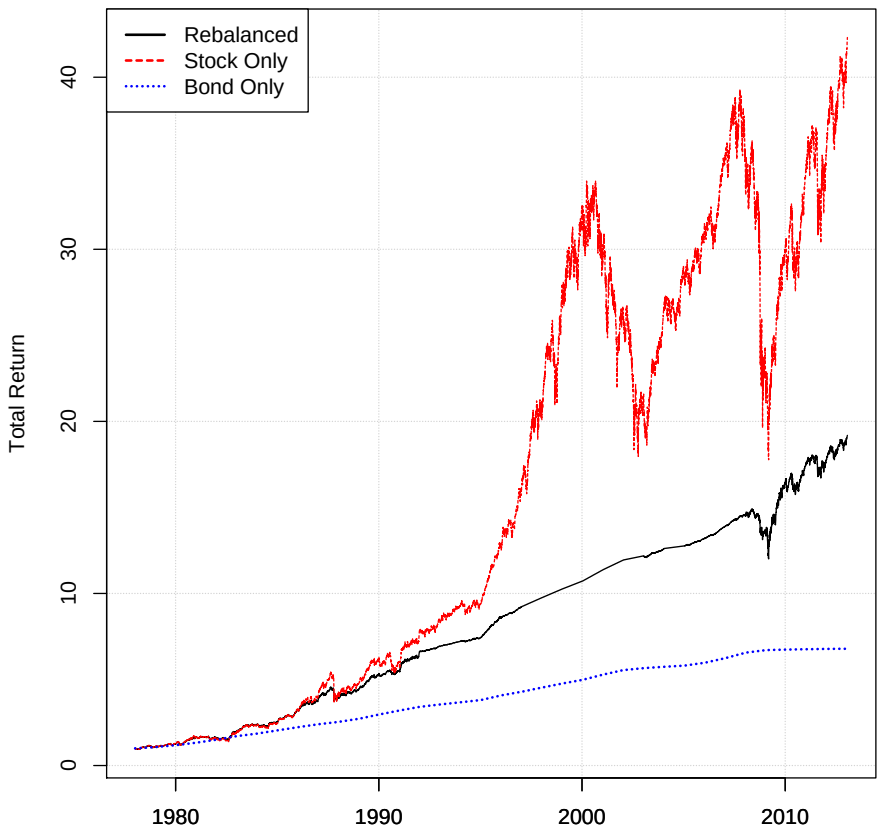
This can also be shown in a table...

Medium Risk Rebal. – Annualized Return (1978-2013)

Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(13.9%)	3.9%	6.0%	9.3%	12.8%	65.6%	9.7%	0.06	0.18	0.74
2	(4.3%)	4.8%	6.7%	9.0%	11.9%	33.0%	6.2%	0.02	0.09	0.68
3	(1.1%)	5.0%	7.5%	8.9%	12.2%	27.5%	5.4%	0.002	0.07	0.66
4	(0.004%)	5.0%	7.2%	8.8%	12.1%	27.8%	5.1%	0.0001	0.05	0.62
5	0.2%	4.9%	6.9%	8.8%	13.0%	26.9%	5.1%	0	0.03	0.60
6	1.6%	4.8%	6.8%	8.7%	13.5%	22.2%	4.8%	0	0.03	0.62
7	1.6%	4.9%	6.8%	8.6%	12.9%	22.5%	4.7%	0	0.03	0.69
8	2.0%	4.8%	6.6%	8.6%	12.9%	20.3%	4.6%	0	0.03	0.78
9	2.5%	4.8%	6.6%	8.4%	12.3%	19.3%	4.3%	0	0.03	0.84
10	2.7%	4.9%	6.8%	8.3%	11.7%	18.1%	4.1%	0	0.03	0.81

Example: Investing for 2 years had mean annualized return 9.0%, min (4.3%), max 33.0%, stdev 6.2%. Investing for 10 years had mean 8.3%, min 2.7%, max 18.1%.

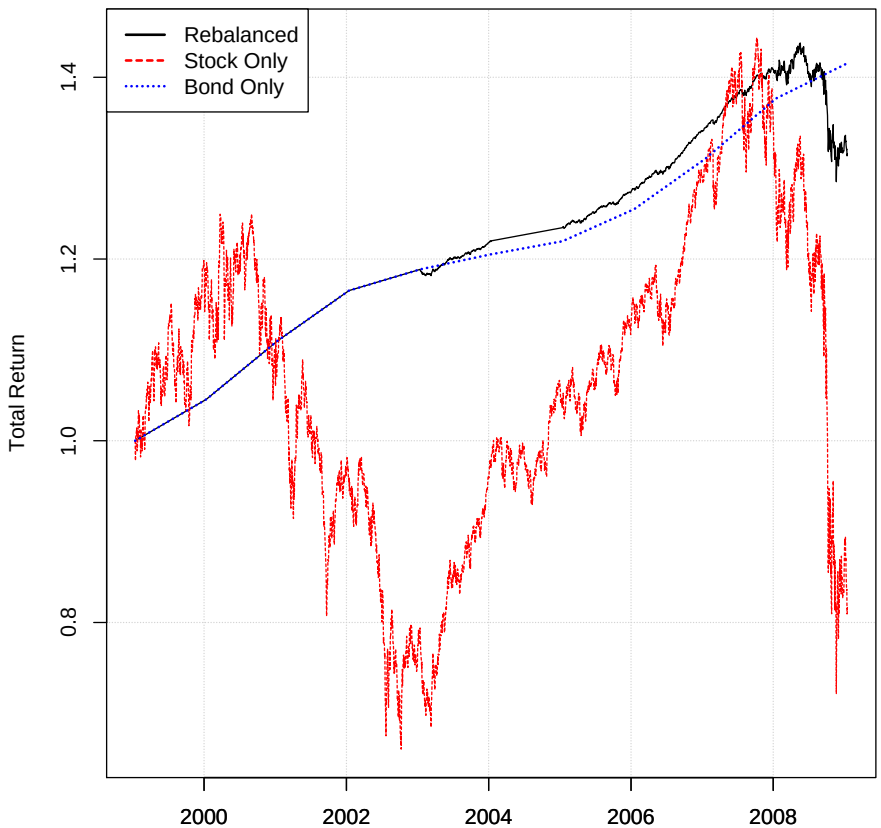
Long-Term Relative Performance (1978-2013)



In this 35 year period the Medium Risk adaptive rebalancing performed better than US Gov. Bonds but worse than S&P 500.

But this is not always the case ...

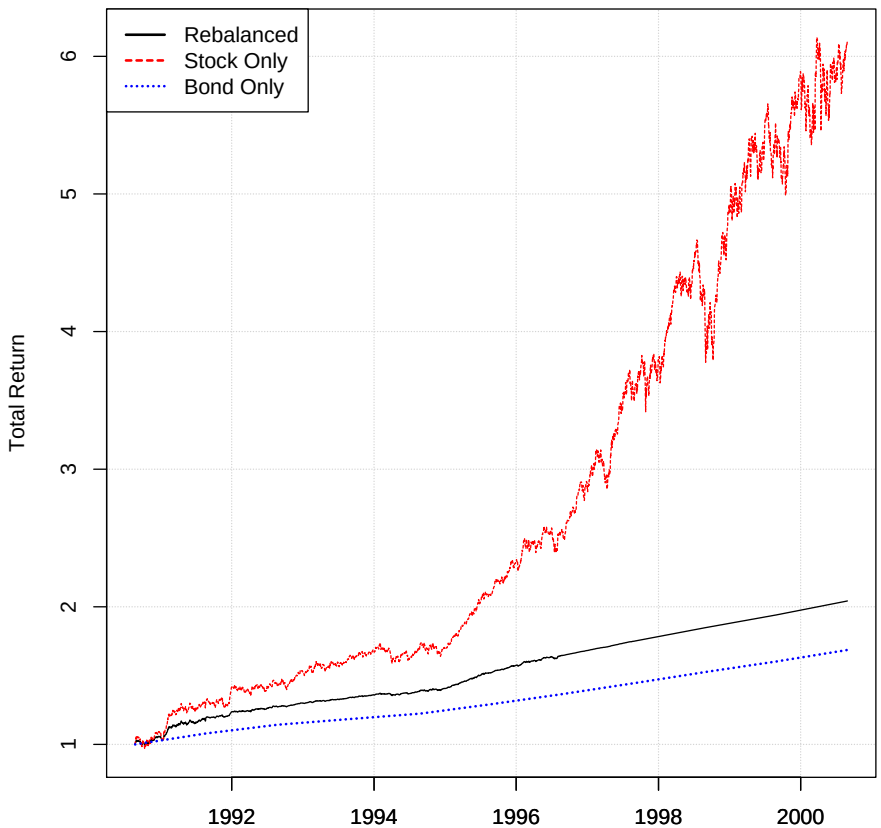
Example: Rebalancing is BETTER Than S&P 500



Example of Medium Risk adaptive rebalancing performing better than S&P 500 and worse than US Gov. Bonds.

Investment period is 10 years.
Starting date is January 12, 1999.

Example: Rebalancing is WORSE Than S&P 500



Example of Medium Risk adaptive rebalancing performing worse than S&P 500 and better than US Gov. Bonds.

Investment period is 10 years.
Starting date is August 23, 1990.

Probability of Under-Performance

Medium Risk Adaptive Rebalancing				
Years of Investing	(...)	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(...)	0.06	0.18	0.74
2		0.02	0.09	0.68
3		0.002	0.07	0.66
4		0.0001	0.05	0.62
5		0	0.03	0.60
6		0	0.03	0.62
7		0	0.03	0.69
8		0	0.03	0.78
9		0	0.03	0.84
10		0	0.03	0.81

- These are historical probabilities (frequencies) for 1978-2013.
- Probability of loss decreases with longer investment duration.
- Probability of under-performing US Gov. Bonds decreases with longer investment duration.
- Probability of under-performing S&P 500 is high at 0.60-0.84.

Conclusion

- Adaptive rebalancing has several advantages over fixed rebalancing for similar levels of mean annualized return:
- Adaptive rebalancing had much lower probability and magnitude of loss.
- ... and significantly lower probability of under-performing S&P 500 and US Gov. Bonds.

The book also studies other adaptive strategies.